Energy Loss and Gluonic Field Distribution to All Orders in 1/N

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arXiv: 1106.5418 [hep-th], with B. Fiol
arXiv: 1112.2345 [hep-th], with B. Fiol and A. Lewkowycz

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What?

Energy radiated by an infinitely massive half-BPS point particle charged under $\mathcal{N} = 4$ $SU(N)$ SYM, symmetric or antisymmetric representations, $T = 0$, large N, λ (AdS/CFT).

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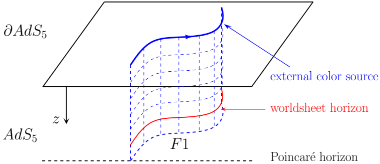
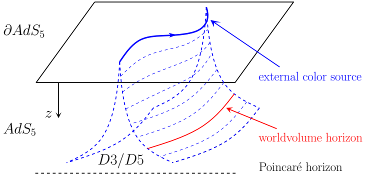
k-symmetric rep.	$\longleftrightarrow$	D3 $(AdS_2 \times S^2 \hookrightarrow AdS_5)$
k-antisymmetric rep.	$\longleftrightarrow$	D5 $(AdS_2 \times S^4 \hookrightarrow AdS_5 \times S^5)$





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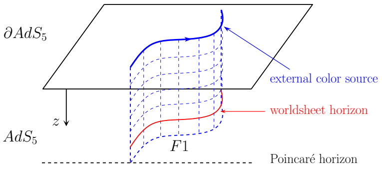

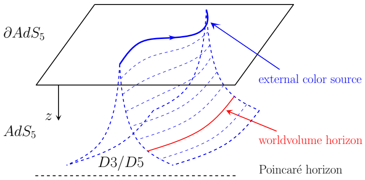

Why?

QGP, Unruh effect, all-orders series in 1/N

Fundamental representation $\longleftrightarrow$ F1 ($AdS_2 \hookrightarrow AdS_5$)

- Wilson loops [Rey-Yee ’98; Maldacena ’98; Drukker-Gross-Ooguri ’99]
- $T = 0$, arbitrary time-like trajectory at $\partial AdS \to F1$ solving NG
  - Energy loss by radiation: [Mikhailov ’03]
  - $\langle T_{\mu\nu} \rangle$: [Athanasiou et al. ’10; Hatta et al. ’11]
  - $\langle \mathcal{L} \rangle$: [Callan-Güijosa ’99; Chernicoff-Güijosa-Pedraza ’11]
- $T \neq 0$, constant velocity at $\partial AdS \to$ trailing string
  - Drag force and energy loss: [Gubser ’06; Herzog et al. ’06]

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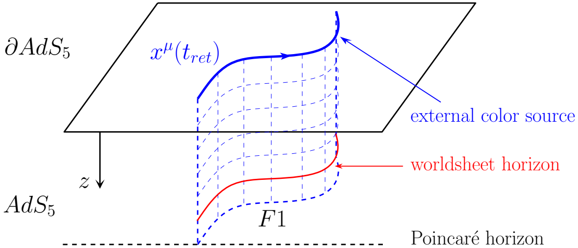
Fundamental representation $\longleftrightarrow$ F1 ($AdS_2 \hookrightarrow AdS_5$)

Energy loss (arbitrary motion):

$$\vec{x}(t_{ret}, z) = \vec{x}(t_{ret}) + z\gamma\vec{v}(t_{ret}) \quad ; \quad t = t_{ret} + \gamma z$$

$$E_{_F} = \frac{\sqrt{\lambda}}{2\pi}\left(\int dt_{ret}\, a^\mu a_\mu + \gamma\frac{1}{z}|_{z=0}\right) \quad \rightarrow \quad \boxed{P_{_F} = \frac{\sqrt{\lambda}}{2\pi} a^\mu a_\mu}$$

[Mikhailov '03]

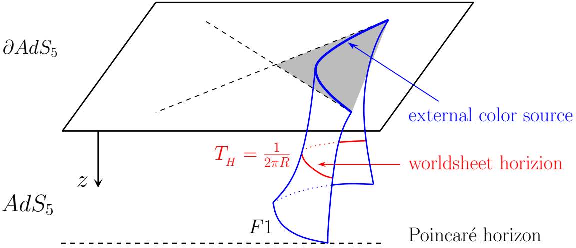


Fundamental representation $\longleftrightarrow$ F1 $(AdS_2 \hookrightarrow AdS_5)$

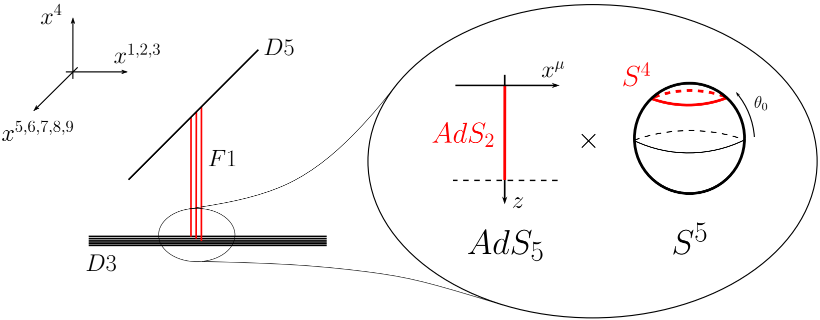
Energy loss (1D constant proper acceleration): $\gamma^3 a = 1/R$

$$-(x^0)^2 + (x^1)^2 = R^2 \quad ; \quad z^2 = R^2 + (x^0)^2 - (x^1)^2$$

$$E_{_F} = \int_{\partial AdS}^{horizon} \mathcal{E}_{_F}\, dz = \frac{\sqrt{\lambda}}{2\pi}\left(\frac{-x^0}{R^2} + \gamma \frac{1}{z}|_{z=0}\right) \rightarrow \boxed{P_{_F} = \frac{\sqrt{\lambda}}{2\pi}\frac{1}{R^2}}$$



Antisymmetric rep. $\longleftrightarrow$ D5 ($AdS_2 \times S^4 \hookrightarrow AdS_5 \times S^5$)



Wilson loops [Yamaguchi '06; Gomis-Passerini '06]

F1: $\Sigma \hookrightarrow M$
D5: $\Sigma \times S^4 \hookrightarrow M \times S^5$ | $\sin\theta_0 \cos\theta_0 - \theta_0 = \pi \left(\frac{k}{N} - 1\right)$ [Hartnoll '06]

Energy loss and drag force ($T \neq 0$) [Chernicoff-Güijosa '06]

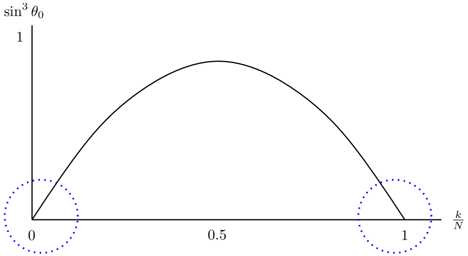
Antisymmetric rep. $\longleftrightarrow$ D5 $(AdS_2 \times S^4 \hookrightarrow AdS_5 \times S^5)$

Energy loss (arbitrary trajectory): Hartnoll + Mikhailov = $\mathcal{E}_{D5}$

$$E_{D5} = \int d^5x \; \mathcal{E}_{D5} \quad \longrightarrow \quad \boxed{P_{A_k} = \frac{2N}{3\pi} \sin^3\theta_0 P_F}$$

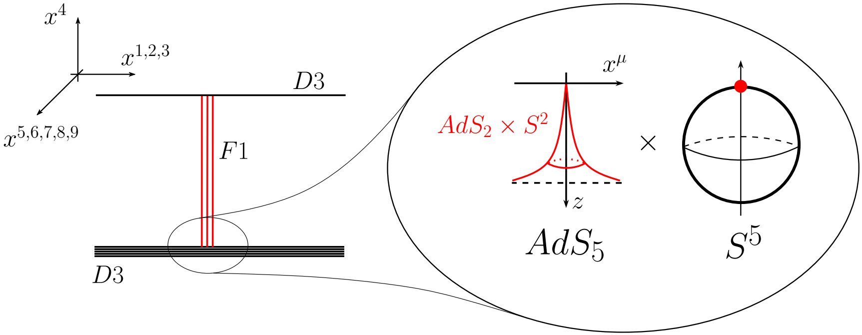
[Fiol-Garolera '11]

$$\boxed{e^2_{A_k} \xrightarrow{\text{strong coupling}} \frac{\sqrt{\lambda}}{2\pi^2} N \sin^3\theta_0}$$



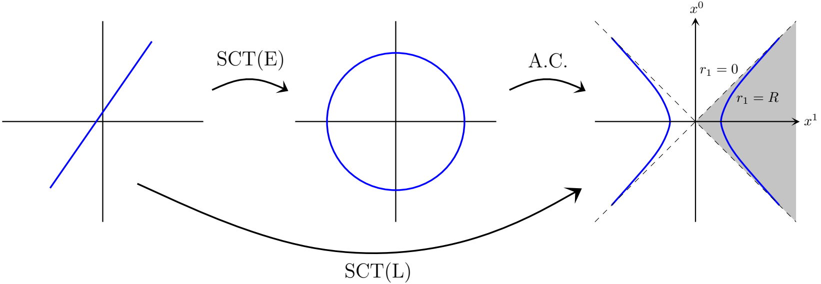
$$\underbrace{\frac{N^2}{\lambda^2} >> \overbrace{N \sin^3\theta_0}^{\text{SUGRA approx.}}}_{\text{probe approx.}} \overbrace{>> \frac{N}{\lambda^{3/4}}}$$

Symmetric rep. $\longleftrightarrow$ D3 $(AdS_2 \times S^2 \hookrightarrow AdS_5)$



Wilson loops [Rey-Yee '98; Drukker-Fiol '05; Gomis-Passerini '06]

Symmetric rep. $\longleftrightarrow$ D3 $(AdS_2 \times S^2 \hookrightarrow AdS_5)$



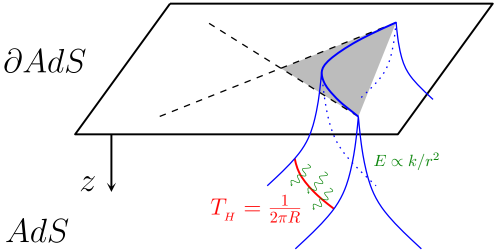
Symmetric rep. $\longleftrightarrow$ D3 $(AdS_2 \times S^2 \hookrightarrow AdS_5)$

Energy loss (CPA):

$$\mathcal{E}_{D3} \longrightarrow E_{D3} = \int_{horizon}^{\partial AdS} d^3x\, \mathcal{E}_{D3} = \frac{2N\kappa}{\pi}\left(\frac{-x^0}{R^2}\sqrt{1+\kappa^2} + \gamma\frac{1}{z}|_{z=0}\right)$$

$$P_{Sk} = \frac{k\sqrt{\lambda}}{2\pi}\sqrt{1+\frac{k^2\lambda}{16N}\frac{1}{R^2}} \longrightarrow \boxed{P_{S_k} = \frac{k\sqrt{\lambda}}{2\pi}\sqrt{1+\frac{k^2\lambda}{16N}a^\mu a_\mu}}$$

[Fiol-Garolera '11]



$$\underbrace{\frac{N^2}{\lambda^2} >> k}_{\text{probe approx.}} \overbrace{>> \frac{N}{\lambda^{3/4}}}^{\text{SUGRA approx.}}$$

Gluonic fields of a static particle to all orders in 1/N

$$\mathcal{O}_{F^2} = \frac{1}{2g_{YM}^2} \mathrm{Tr}\,\left(F^2 + [X_I, X_J][X^I, X^J] + \text{fermions}\right) \quad \longleftrightarrow \quad \phi$$

$$\langle \mathcal{O}_{F^2} \rangle = \frac{f(k,\lambda,N)}{|\vec{x}|^4}$$

$$\downarrow$$

$$\langle \mathcal{O}_{F^2} \rangle_{\text{fund}} = \frac{\sqrt{\lambda}}{16\pi^2} \frac{1}{|\vec{x}|^4}$$ [Danielsson et al. ’99; Callan-Güijosa ’00]

$$\langle \mathcal{O}_{F^2} \rangle_{S_k} = \frac{k\sqrt{\lambda}}{16\pi^2} \frac{\sqrt{1+\frac{k^2\lambda}{16N^2}}}{|\vec{x}|^4}$$ [Fiol-Garolera-Lewkowycz ’11]

THE SAME RESULT!

Conclusions and further research

- WS/WV horizon $\longleftrightarrow$ energy loss by radiation

- Effective chromo-electric charges?

$$e^2_{A_k} \to \frac{\sqrt{\lambda}}{2\pi^2} N \sin^3\theta_0 \qquad\qquad e^2_{S_k} \to \frac{3k\sqrt{\lambda}}{4\pi}\sqrt{1+\frac{k^2\lambda}{16N^2}}$$

- $\langle T_{\mu\nu}\rangle_{S_k}$(static particle) $\sim$ supersymmetry arguments

- All correction in $1/N$? $\to$ Matrix Model

- Angular distribution of radiated power